Tutorial on Circuit (Part 2)
Introduction to Electrical and Electronic Engineering

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Overview

* **Learning Objectives:**
  Analyze circuits with ideal operational amplifiers
* **Basics**
  - Symbols & Rules
  - Operational Amplifiers
    - The Ideal op-amp Model
    - Ideal op-amp in a negative feedback configuration
* **Questions & Summary**

Symbols & Rules(Recap)

- **Operational Amplifiers**

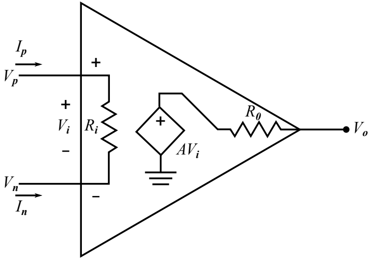


Figure : **Equivalent circuit model of op-amp device**

In the absence of any load at the output, the output voltage is

$$V_o = AV_i = A(V_p - V_n)$$

Operational Amplifiers(Recap)

- **Operational Amplifiers**

  In the absence of any load at the output, the output voltage is
  $$V_o = AV_i = A(V_p - V_n)$$

Which indicates that the output voltage V_o is a function of the difference between the input voltages V_p and V_n. For this reason op-amps are **difference amplifiers**.

- **The Ideal Op-amp Model**

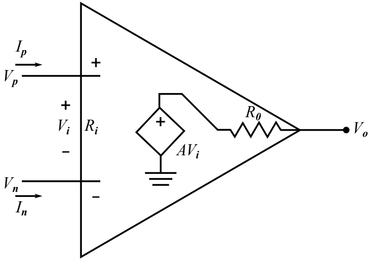


Figure : **Ideal op-amp model**

Operational Amplifiers(Recap)

An ideal op-amp is a device which acts as an ideal voltage controlled voltage source. Referring to Figure in the previous slide, this implies that the device will have the following characteristics:

* **No current flows into the input terminals** of the device. This is equivalent to having an infinite input resistance $R_i = \infty$. In practical terms this implies that the amplifier device will make no power demands on the input signal source.
* **Have a zero output resistance** $(R_o = 0)$. This implies that the output voltage is independent of the load connected to the output.

- **The Ideal Op-amp Model**

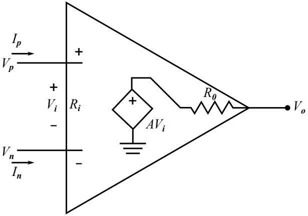


Figure : **Ideal op-amp model**

* In summary, the ideal op-amp conditions are:

  - $I_p = I_n = 0$ **No current into the input terminals**
  - $R_i \to \infty$ **Infinite input resistance**
  - $R_o = 0$ **Zero output resistance**
  - $A \to \infty$ **Infinite open loop gain**

Operational Amplifiers(Recap)

- When an op-amp is arranged with a **negative feedback** the ideal rules are:

* $I_p = I_n = 0$: input current constraint
* $V_n = V_p$: input voltage constraint

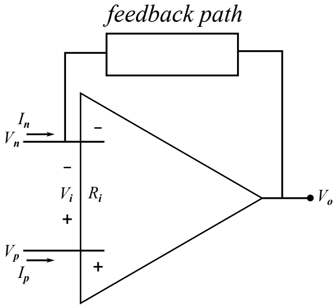
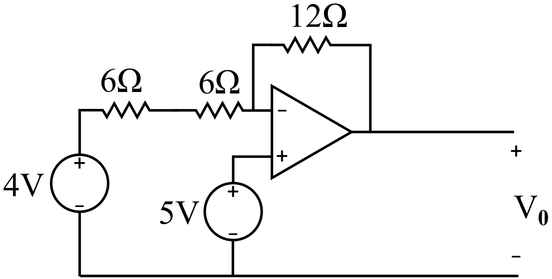


Figure : **Basic negative feedback configuration.**

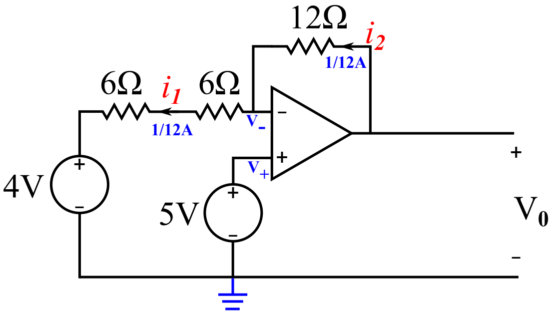
Question 1

* Determine V_0 in the following circuit. Assume that the op-amp is ideal.



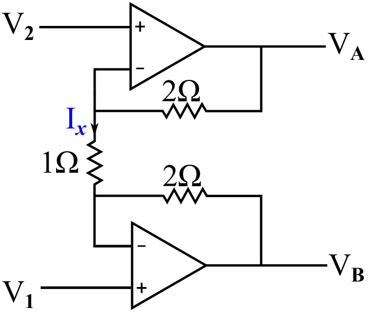
Solution(Q1)

* Since $V_- = V_+$, $V- = 5V$. So there must be 1/12A flowing left through the two 6Ω resistors($i_1 = \frac{1}{12}A$). There must be a corresponding 1/12A($i_2 = \frac{1}{12}A$) flowing to the left through the 12 ohm resistor. V_0 is then the sum of $V_- = 5V$ and the 1V across the 12Ω resistor.



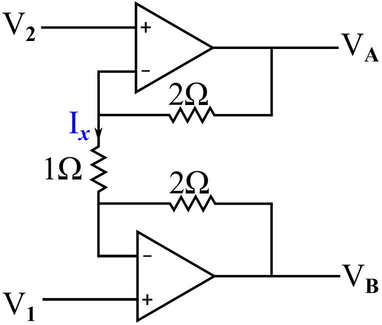
Question 2

- Determine the current I_x when $V_1 = 1V$ and $V_2 = 2V$.
- Determine the voltage V_A when $V_1 = 1V$ and $V_2 = 2V$.
- Determine a general expression for V_A in terms of V_1 and V_2.



Solution(Q2)

- Determine the current I_x when $V_1 = 1V$ and $V_2 = 2V$.
- Determine the voltage V_A when $V_1 = 1V$ and $V_2 = 2V$.
- Determine a general expression for V_A in terms of V_1 and V_2.

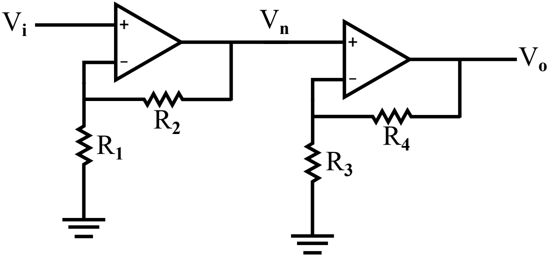


- When $V_1 = 1V$ and $V_2 = 2V$, $I_x = 1A$
- When $V_1 = 1V$ and $V_2 = 2V$, $V_A = 4V$
- A general expression for V_A:

$$\begin{aligned} V_A &= V_2 + I_x \times 2 \\ &= V_2 + \frac{V_2 - V_1}{1} \times 2 \\ &= V_2 + 2(V_2 - V_1) \\ &= -2V_1 + 3V_2 \end{aligned}$$

Question 3

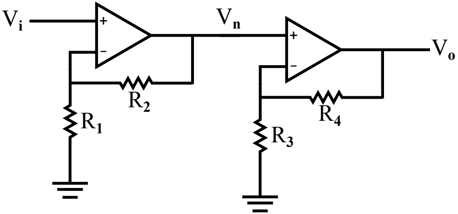
- Use a single op-amp and resistors to make a circuit that is equivalent to the following circuit.

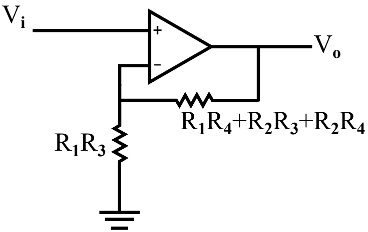


Solution(Q3)

* Use a single op-amp and resistors to make a circuit that is equivalent to the following circuit.

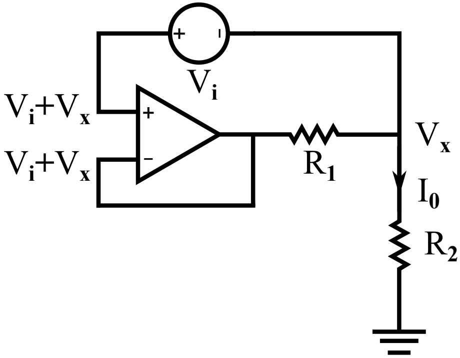
$$\frac{V_n}{V_i} = (1 + \frac{R_2}{R_1})$$
$$\frac{V_0}{V_i} = \frac{V_0}{V_n}\frac{V_n}{V_i} = (1 + \frac{R_3}{R_4})(1 + \frac{R_2}{R_1})$$
$$1 + \frac{R_1 R_4 + R_2 R_3 + R_2 R_4}{R_1 R_3}$$





Question 4

- Use the ideal op-amp model($V_+ = V_-$)to determine an expression for the output current I_0 in terms of the input voltage V_i and resistors R_1 and R_2.



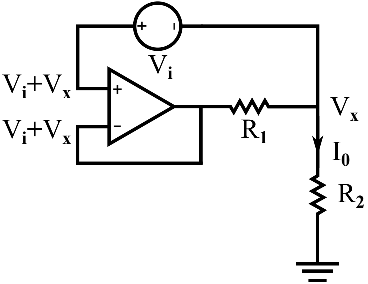
Solution(Q4)

* Use the ideal op-amp model($V_+ = V_-$)to determine an expression for the output current I_0 in terms of the input voltage V_i and resistors R_1 and R_2.

$$V_x = (V_i + V_x)\frac{R_2}{R_1+R_2}$$

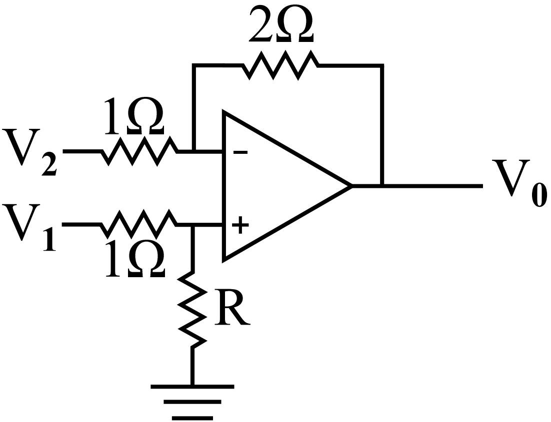
$$\Rightarrow \quad V_x = V_i\frac{R_2}{R_1}$$

$$I_0 = \frac{V_x}{R_2} = \frac{1}{R_2}V_i\frac{R_2}{R_1} = \frac{V_i}{R_1}$$

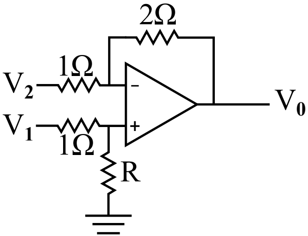


Question 5

* Determine R so that $V_0 = 2(V_1 - V_2)$.



Solution(Q5)

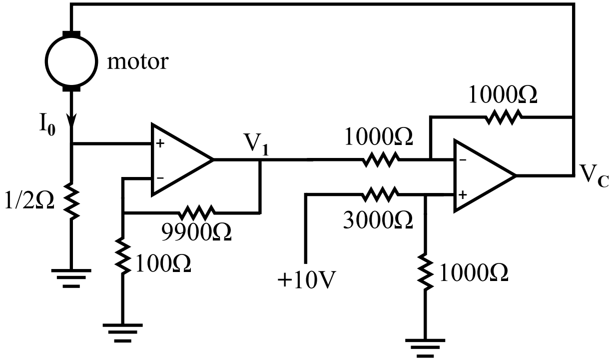


* **No current in V_- or V_+ inputs:**
  $V_+ = \frac{R}{1+R} V_1$
  $V_- = V_2 + \frac{1}{1+2}(V_0 - V_2) = \frac{2}{3} V_2 + \frac{1}{3} V_0$

* **Ideal op-amp:**
  $V_+ = V_- \Rightarrow \frac{R}{1+R} V_1 = \frac{2}{3} V_2 + \frac{1}{3} V_0$
  $V_0 = \frac{3R}{1+R} V_1 - 2V_2 \quad \rightarrow \quad \frac{3R}{1+R} = 2 \quad \rightarrow \quad R = 2\Omega$

- A proportional controller that regulates the current through a motor by setting the motor voltage V_C to $V_C = K(I_d - I_0)$

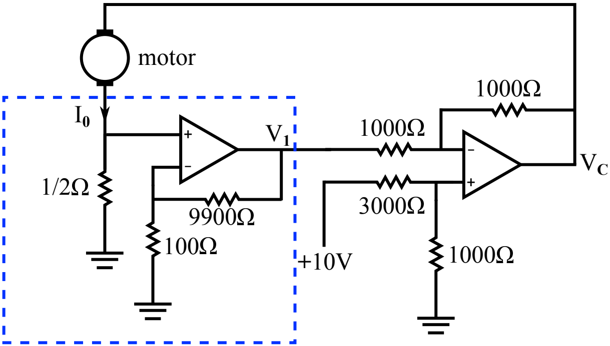
- K is the gain(ohms)
- I_d is the desired motor current
- I_0 is the actual current through the motor.



Question 6

* A proportional controller that regulates the current through a motor by setting the motor voltage V_C to $V_C = K(I_d - I_0)$

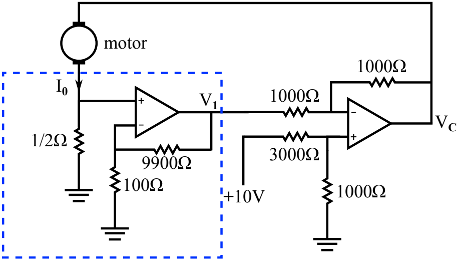
* Consider the circuit inside the dotted rectangle. Determine V_1 as a function of I_o
* Determine the gain K and desired motor current I_d



Solution(Q6)

- Consider the circuit inside the dotted rectangle. Determine V_1 as a function of I_0.
- $V_+ = 1/2xI_0 = V_-$
  $V_- = 100/(100 + 9900) \times V_1$

$\Rightarrow$ $V_1 = 1/2xI_0 \times 100$

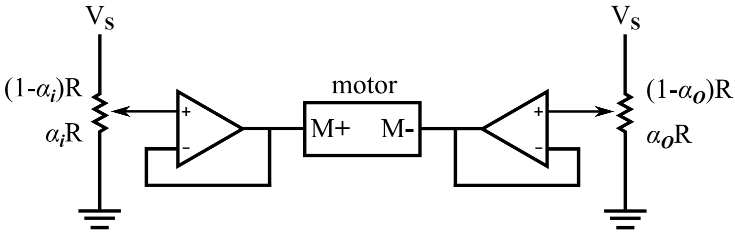


- Determine the gain K and desired motor current I_d.
- KCL at V_- input to right op-amp:

$$\frac{V_c - 2.5}{1000} = \frac{2.5 - V_1}{1000} \Rightarrow V_c = 50(0.1 - I_0)$$

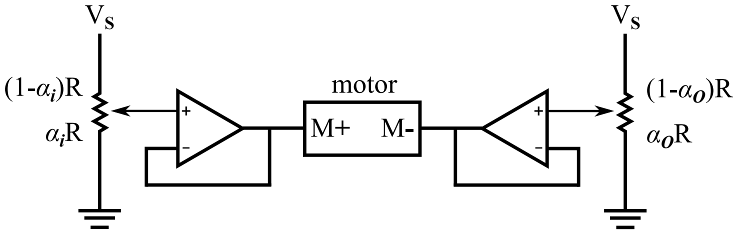
Question 7

* The following figure shows a motor controller. A **human** can turn the **left potentiometer** (the input pot). Then the **motor** will turn the **right potentiometer** (the output pot) so that the **shaft angle of the output pot tracks that of the input pot.**



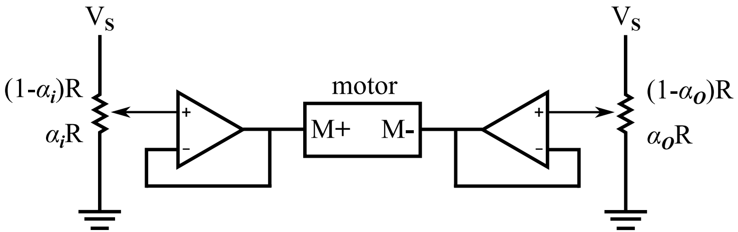
Question 7

* Pot resistances depends on shaft angle
  - Lower part of the pot is αR
  - Upper part is $(1-\alpha)R$, where $R = 1000\Omega$
  - α is from 0 (most counterclockwise position) to 1 (most clockwise position)
* If $\alpha_i > \alpha_o$, then the voltage to the motor ($V_{M+} - V_{M-}$) is positive, and the motor turns clockwise (so as to increase α_o) – i.e., **positive motor voltage $\rightarrow$ clockwise rotation.**



Question 7(a)

- Determine an expression for V_{M+} in terms of α_i, R, and V_s.

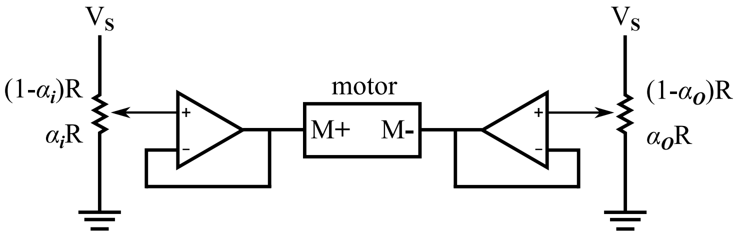


Solution(Q7(a))

* Determine an expression for V_{M+} in terms of α_i, R, and V_S.
* The output of the voltage divider is

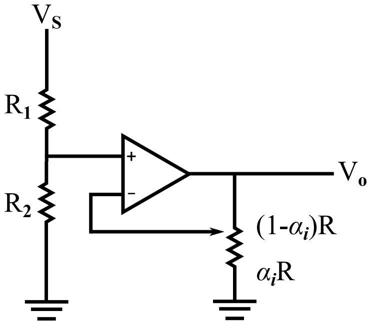
$$V_+ = \frac{\alpha_i R}{\alpha_i R + (1-\alpha_i)R} V_S = \alpha_i V_S$$

* The op-amp provides a gain of 1, so $V_{M+} = V_+$.



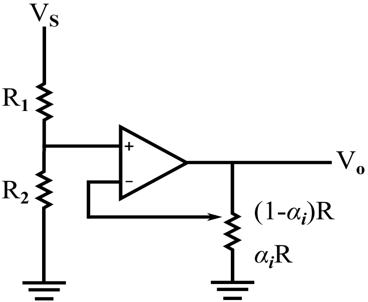
Question 7(b)

* The following circuit produces a voltage V_o that depends on the position of the input pot. Determine an expression for the voltage V_o in terms of α_i, R, R_1, R_2, and V_s.



Solution(Q7(b))

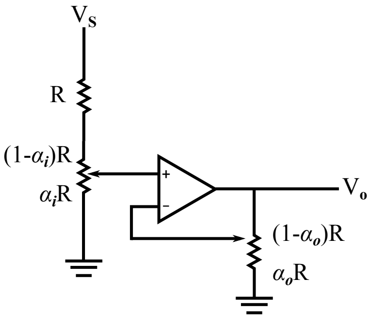
- The following circuit produces a voltage V_o that depends on the position of the input pot. **Determine an expression for the voltage V_o in terms of α_i, R, R_1, R_2 and V_s.**



- The positive input to the op-amp is connected to a voltage divider with equal resistors so
  $V_+ = \frac{R_2}{R_1+R_2} V_s$
- The input pot is on the output of the op-amp, so
  $V_- = \frac{\alpha_i R}{\alpha_i R+(1-\alpha_i)R} V_o = \alpha_i V_o$
- In an ideal op-amp, $V_+ = V_-$ so $V_o = \frac{R_2 V_s}{(R_1+R_2)\alpha_i}$

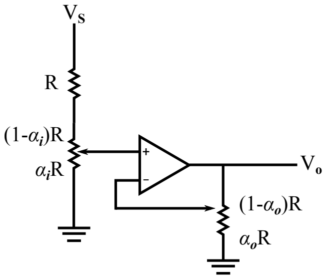
Question 7(c)

- The following circuit produces a voltage V_o that depends on the positions of both pots.
  Determine an expression for V_o in terms of α_i, α_o, R, and V_s



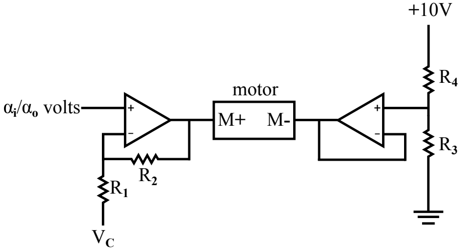
Solution(Q7(c))

* The following circuit produces a voltage V_0 that depends on the position of the input pot. Determine an expression for the voltage V_o in terms of α_i, α_o, R and V_s.
* The positive input to the op-amp is connected to pot 1 so that $V_+ = \frac{\alpha_i R}{\alpha_i R + (1-\alpha_i)R + R} V_s = \frac{\alpha_i V_s}{2}$
* The output pot is on the output of the op-amp, so $V_- = \frac{\alpha_o R}{\alpha_o R + (1-\alpha_o)R} V_o = \alpha_o V_o$
* In an ideal op-amp, $V_+ = V_-$ so $V_o = \frac{\alpha_i}{\alpha_o} \frac{V_s}{2}$



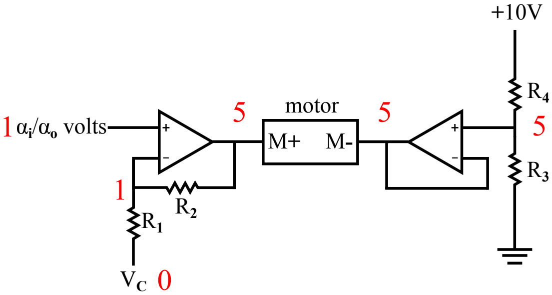
Question 7(d)

- Assume that we are provided with a circuit whose output is α_i/α_o volts. We want to **design a motor controller** of the following form so that the motor shaft angle (which is proportional to α_o) will track the input pot angle (which is proportional to α_i).
- Assume that $R_1 = R_3 = R_4 = 1000\Omega$ and $V_C = 0$. Is it possible to choose R_2 so that α_o tracks α_i? If **yes**, enter an acceptable value for R_2.



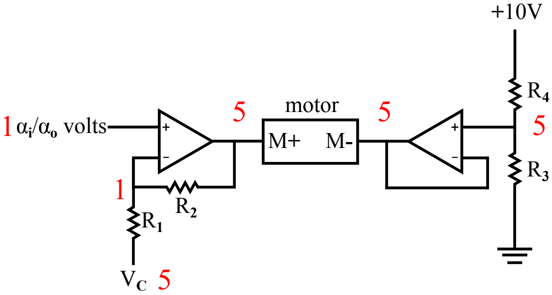
Solution(Q7(d))

- Assume that $R_1 = R_3 = R_4 = 1000\Omega$ and $V_C = 0$
- If $R_3 = R_4$ then the right motor input is 5V. If $\alpha_i = \alpha_o$then the gain of the **left** op-amp circuit must be **5** so that the motor voltage is 0. The gain is $R_1 + R_2/R_1$, so R_2 must be 4000Ω.



Solution(Q7(d))

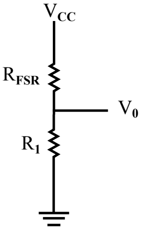
* Assume that $R_1 = R_3 = R_4 = 1000\Omega$ and $V_C = 5V$
* If $R_3 = R_4$ then the right motor input is 5V. If $\alpha_i = \alpha_o$ then $V_+ = V_- = 1$ for the right op-amp. We need the left motor input to be 5V. But if the left motor input is 5V and $V_C = 5V$ then V_- must also be 5V, which leads to a contradiction.



Question 8(a)

- You have to design a hammer machine(i.e. using a hammer to hit a platform to see how strong the participants are). The design goal is to generate an output voltage (V_o) which is proportional to the force(F) applied on the hammer, i.e. $V_o = m \times F + C(m > 0 \text{ and } C > 0)$.
- (a) You found a force-sensitive resistor (FSR) from the catalog, which can be modeled by $R_{FSR} = 10k\Omega/F$

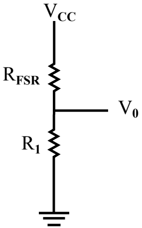
- You then design a circuit as a potential divider. Will this circuit correctly implement?



Solution(Q8(a))

- You have to design a hammer machine(i.e. using a hammer to hit a platform to see how strong the participants are). The design goal is to generate an output voltage (V_o) which is proportional to the force(F) applied on the hammer, i.e. $V_o = m \times F + C$, with $m > 0$ and $C > 0$.
- (a) You found a force-sensitive resistor (FSR) from the catalog, which can be modeled by $R_{FSR} = 10k\Omega / F$

You then design a circuit as a potential divider. Will this circuit correctly implement?

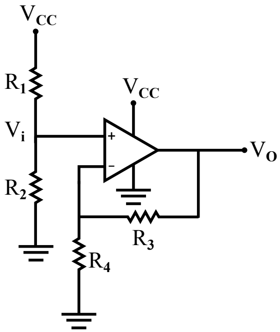


A : **No**, because V_o is not linearly proportional to F.

$(V_o = V_{cc} \cdot \frac{R_1}{R_1 + R_{FSR}} = V_{cc} \cdot \frac{R_1}{R_1 + \frac{10k\Omega}{F}})$

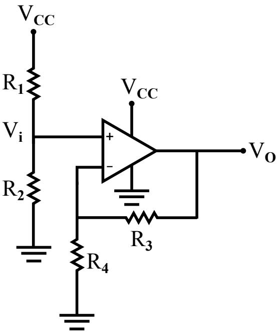
Question 8(b)

∗ (b) Find the gain of the following circuit:



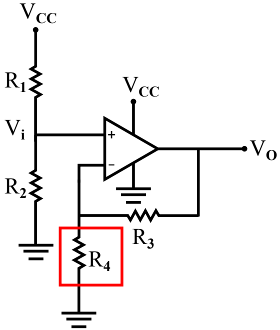
Solution(Q8(b))

* (b) Find the gain of the following circuit:
* At the two op-amp inputs, $V_- = V_+ = V_i$.
* Since V_i is related to V_o through the two resistors such that
  $V_i = V_- = \frac{R_4}{R_3+R_4} V_o$,
  $\Rightarrow V_o = (1 + \frac{R_3}{R_4}) V_i$



Question 8(c)

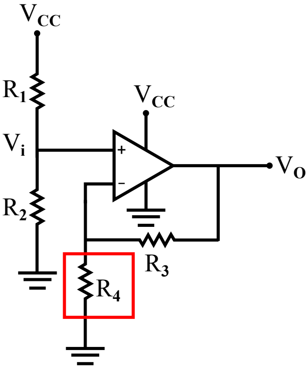
* (c) Design(by using the **non-inverting** amplifier circuit) a circuit such that the output voltage(V_o) is directly proportional to the input force(F).



Solution(Q8(c))

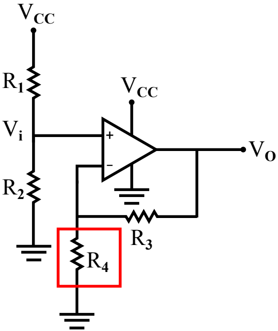
* (c) Design (by using the **non-inverting** amplifier circuit) a circuit such that the output voltage (V_o) is directly proportional to the input force(F).

* Replace R_4 by the FSR. We then have
  $V_o = (1 + \frac{R_3 F}{10000})V_i = \frac{R_3 V_i}{10000}F + V_i$
  V_o is a linear function of F



Question 8(d)

- (d) The system requires that
  - when the force $F = 0N$, the output voltage $V_o = 4V$
  - when $F = 20N$, $V_o = 12V$
  - Construct the circuit designed in (c) using only **one FSR**, **one op-amp**, **one** $12V$ **power supply**, and an **unlimited** number of $1k\Omega$ **resistors**.



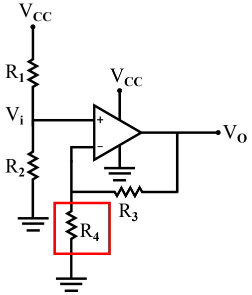
Solution(Q8(d))

$$V_o = (1 + \frac{R_3 F}{10000})V_i = \frac{R_3 V_i}{10000}F + V_i$$

(d) The system requires that

when the force $F = 0N$, $V_o = 4V$

when $F = 20N$, $V_o = 12V$



* Construct the circuit designed in (c) using only one FSR, one op-amp, one 12V power supply, and 1k ohm resis.
* When $F = 0N$, $V_o = V_i = 4V$. We can use $R_1 = 2k\Omega$ and $R_2 = 1k\Omega$($2k\Omega$ resistors can be made by two $1k\Omega$ resistors in series). When F = 20N,

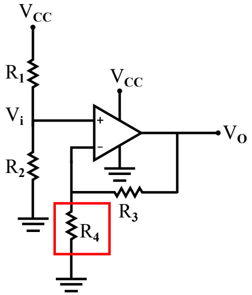
$$12 = \frac{R_3 \times 4}{10000} \times 20 + 4 \Leftarrow R_3 = 1k\Omega$$

Question 8(e&f)

* (e) Using the above circuit, what is the value of V_o when someone hits the hammer too hard, generating a force of $200N$?
* (f) Suggest modification(s) to your answer in Part (d) such that the maximum allowable force to the circuit is $60N$. You can only use the available components in Part(d), while maintaining V_o to be directly proportional to F.

Solution(Q8(e&f))

$$V_o = (1 + \frac{R_3 F}{10000})V_i = \frac{R_3 V_i}{10000}F + V_i$$



(e) Using the above circuit, what is the value of V_o when someone hits the hammer too hard, generating a force of $200N$?

A : **12V**

Notice: the op-amp is working in the saturate state

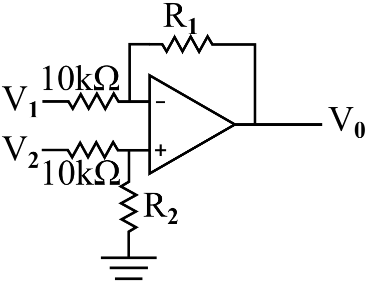
(f) Suggest modification(s) such that the max allowable force to the circuit is $60N$.

A : Change R_3 to $\frac{1}{3}k\Omega$. This can be done by parallel composition of three $1k\Omega$ resistors.

Appendix(Question 9)

* Fill in the values of R_1 and R_2 required to satisfy the equations in the left column of the following table. The values must be non-negative (i.e., in the range $[0, \infty]$)

	R_1	R_2
$V_o = 2V_2 - 2V_1$		
$V_o = V_2 - V_1$		
$V_o = 4V_2 - 2V_1$		



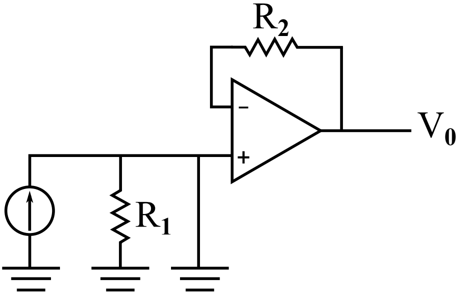
Appendix(Solution(Q9))

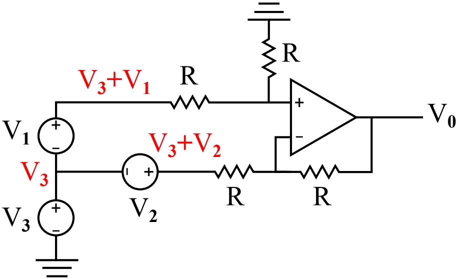
- $V_+ = \frac{R_2}{10k+R_2} V_2 = V_- = \frac{R_1}{10k+R_1} V_1 + \frac{10k}{10k+R_1} V_o$
- $V_o = \frac{10k+R_1}{10k+R_2} \times \frac{R_2}{10k} \times V_2 - \frac{R_1}{10k} \times V_1$

- 3^{rd}: Negative R i.e. Impossible $\Rightarrow$

	R_1	R_2
$V_0 = 2V_2 - 2V_1$	$20k\Omega$	$20k\Omega$
$V_0 = V_2 - V_1$	$10k\Omega$	$10k\Omega$
$V_0 = 4V_2 - 2V_1$	Impossible	Impossible

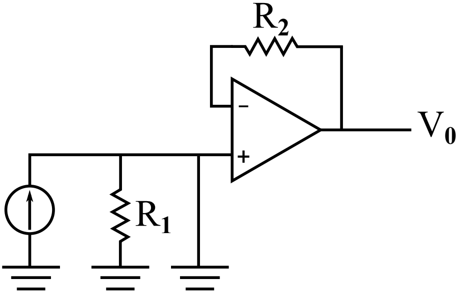
* What is V_o?

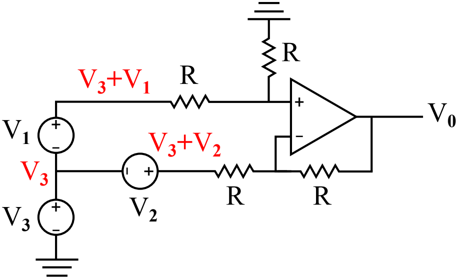




Appendix(Solution(Q10))

* What is V_o?





$V_o = 0$

$V_o = V_1 - V_2$

Appendix(Question 11)

- Students Kim, Pat, Jody, Chris, and Leon are trying to design a controller for a display of three robotic mice in the Rube Goldberg Machine, using a $10V$ power supply and three motors.
    - The first is supposed to spin as fast as possible (in one direction only), the second at half of the speed of the first, and the third at half of the speed of the second.
    - Assume the motors have a resistance of approximately 5Ω and that rotational speed is proportional to voltage.
- For each design, indicate the voltage across each of the motors.

Appendix(Solution(Q11 – Jody's Design))

P.D. of motor 1 $= 10V$

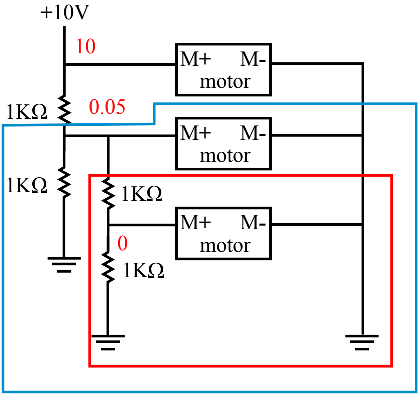
P.D. of motor 2 $= 0.05V$

P.D. of motor 3 $= 0V$

Wrong design

Eq. R.(Red): $1K + \sim 5 \rightarrow 1K$

Eq. R.(Blue): $1K \parallel 1K \parallel 5 \rightarrow \sim 5$



P.D. of motor 1 $= 10V$

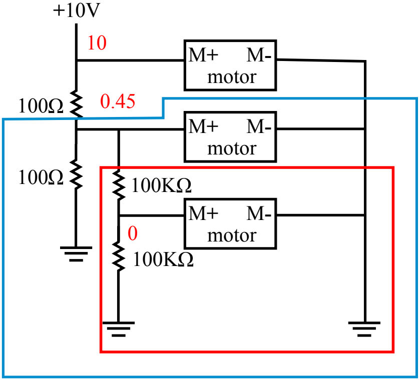
P.D. of motor 2 $= 0.45V$

P.D. of motor 3 $= 0V$

Wrong design

Eq. R.(Red): $100K + \sim 5 \rightarrow 100K$

Eq. R.: $100 \parallel 100K \parallel 5 \rightarrow \sim 5$



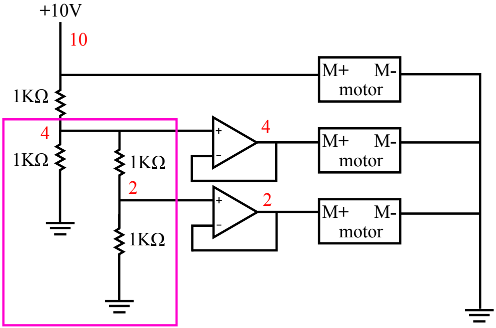
P.D. of motor 1 $= 10V$

P.D. of motor 2 $= 4V$

P.D. of motor 3 $= 2V$

Wrong design

Eq. R.: $1K \parallel 2K = \frac{2}{3}K$



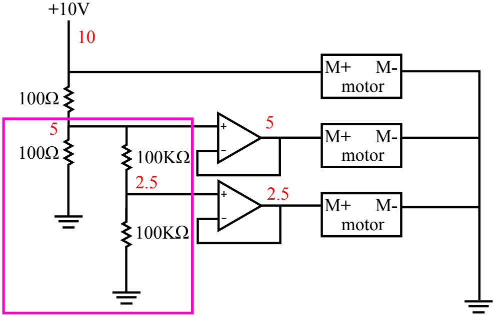
P.D. of motor 1 $= 10V$

P.D. of motor 2 $= 5V$

P.D. of motor 3 $= 2.5V$

Correct design

Eq. R.: $100 \parallel 200K =\sim 100$



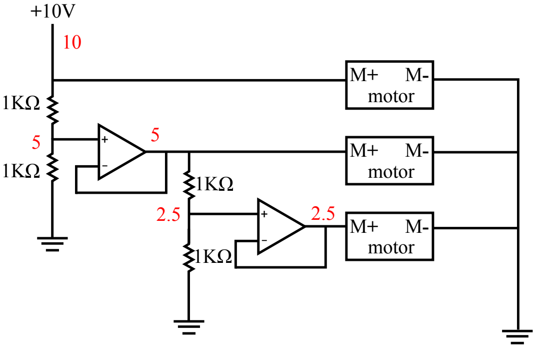
Appendix(Solution(Q11 – Leon's Design))

P.D. of motor 1 $= 10V$

P.D. of motor 2 $= 5V$

P.D. of motor 3 $= 2.5V$

Correct design



Summary

Operational Amplifiers

- Ideal Op-amp Model(Properties)
  - (1) $I_p = I_n = 0$
  - (2) $R_i \to \infty$
  - (3) $R_o = 0$
  - (4) $A \to \infty$
- Ideal op-amp in a negative feedback configuration(conditions)
  - (1) $I_p = I_n = 0$: input current constraint
  - (2) $V_n = V_p$: input voltage constraint

The End